NNSA HSAP Computational Laboratory 5 - 2026 Update

[5] Often in coding you will take an existing program and write a next version with different (extended) capabilities. Did we see that last week (quadratic equation scripts)? As we have mentioned you should never change a script that you have gotten working. Why not? (You might break it!) The right way to make a new version is to make a copy of your script using the *linux* 'cp' command

```
cp geometricsum.py geometricsum2.py
```

[EX: 5-1] We can now edit the copy *geometricsum2.py* to do a more general 'geometric sum'

$$\text{sum} = a(x + x^2 + x^3 + \cdots x^N)$$

Do it!

[6] Geometic sums are sort of cool, especially if you let $N = \infty$.
- Can you figure out a simpler formula for $0.3333333333\cdots$?

[EX: 5-2] Use python in 'interactive mode' to compute 1/3.
Can you figure out a way to prove $0.333333\cdots = \frac{1}{3}$?

[EX: 5-3] **Very hard!** Write $0.4545454545\cdots$ as a fraction.

[7] Let's finish up by returning to 'round-off' errors and learn a bit about how computers store numbers.

Human beings count in 'base 10'. Why?

What does 'base 10' really mean?
- We use ten symbols to write numbers: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
- When we write a number bigger than 9 we need to use multiple symbols.

[EX: 5-4] What does 17 mean? (Explain in words.)

[EX: 5-5] What does 582 mean? (Explain in words.)

[EX: 5-6] What are the biggest numbers you can write using one symbol? Two symbols? Three symbols? Four symbols?
Do you see a way of writing those biggest numbers using powers of 10?

What 'base' do computers use to count? Why?

Let's practice counting in base 2!

[EX: 5-7] Count up to 32 (base 10) in base 2.

[EX: 5-8] **Very, very, very hard!** If you write $0.11\cdots$ in base 2, what is it in base 10?

[8] We will probably skip this. Have you learned about 'special functions' like sine, cosine, exponential, logarithm, etc in your math classes?
One of the most important (and amazing!) facts from calculus is that all the 'special functions' can be written as sums of powers of x !!! This is so bizarre !!!

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots \\ &= 1 + x + \frac{x^2}{1\cdot 2} + \frac{x^3}{1\cdot 2\cdot 3} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} + \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} + \cdots \end{aligned}$$

Notice the *factorials* there!
Similarly

$$\sin(x) = x - \frac{x^3}{1\cdot 2\cdot 3} + \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} + \cdots$$

And

$$\text{cosine}(x) = 1 - \frac{x^2}{1\cdot 2} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} + \cdots$$

It almost looks like exponentials are some combination of sines and cosines, except the minus signs are missing in the exponential.